Satellite Navigation

Global Navigation Satellite System (GNSS)

How could a satellite 20,000 km away possibly tell me where I am?

Bill Hobbs, Member
Modesto PCUG, CA

"I can't say as ever I was lost, but I was bewildered once for three days."
-- *Daniel Boone: The Life and Legend of an American Pioneer*
by John Mack Faragher

Granny Weatherwax was not lost. She wasn’t the kind of person who ever became lost. It was just that, at the moment, while she knew exactly where SHE was, she didn’t know the position of anywhere else.
-- *Wyrd Sisters* by Terry Pratchett

Various and Sundry

Definition of navigation

1. the act or practice of navigating
2. the science of getting ships, aircraft, or spacecraft from place to place; especially : the method of determining position, course, and distance traveled
3. ship traffic or commerce

-- Merriam-Webster.com. Merriam-Webster

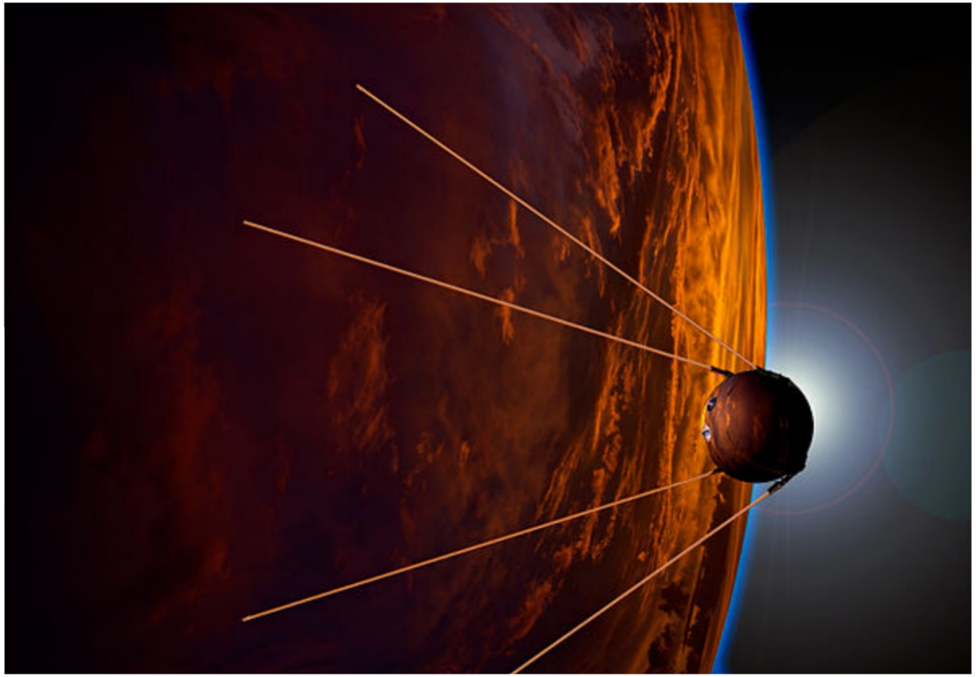
systems

global: US GPS, Russia GLONASS, European Union Galileo, China BeiDou-2

regional: China BeiDou, India IRNSS/NAVIC, Japan QZSS

Birth of Satellite Navigation

1957: Sputnik 1 launched

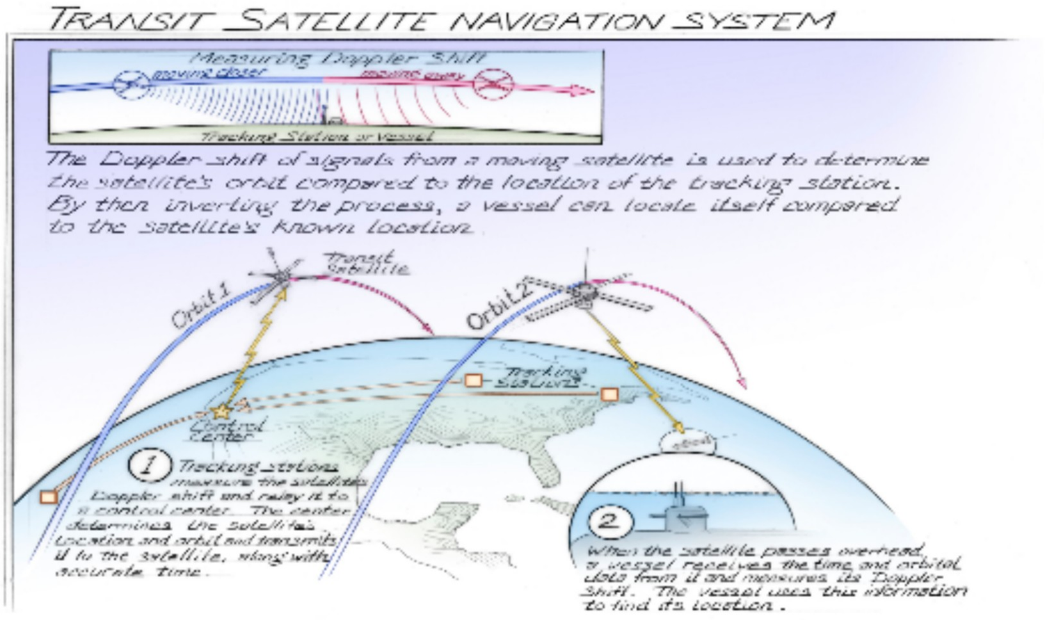


-- Dawn of the Space Age by Gregory R Todd /

Satellite Navigation

- 1958: US Navy begins TRANSIT project
- 1964: TRANSIT becomes operational

Other systems: 621B, Timation, SECOR



-- National Air and Space Museum, Smithsonian Institution, Bruce Morser

Satellite Navigation

- 1973: US begins Defense Navigation Satellite System (DNSS) project, renamed NAVSTAR Global Positioning System (GPS)
- 1976: Soviet Union begins GLONASS project
- 1978: First GPS satellite launch

Satellite Navigation

- 1982: First GLONASS satellite launch
- 1990 - 1991: GPS used during Gulf War
- 1995: GPS and GLONASS fullyoperational
- 2011: Russia restores GLONASS constellation

How Much and Why

- GPS cost USD 10 to 12 billion from start to operational status in 1995
- Primary justification was precision navigation for nuclear forces, there were no civilian considerations
- For the fiscal year ending 2017 Sep 30, GPS operating cost is over USD 900 million

How Much and Why

- In 2013, GNSS services generated an estimated USD 56 billion in US economic benefits:
  - Precision agriculture and earth-moving
  - Surveying
  - Fleet vehicle locations
  - Various consumer applications

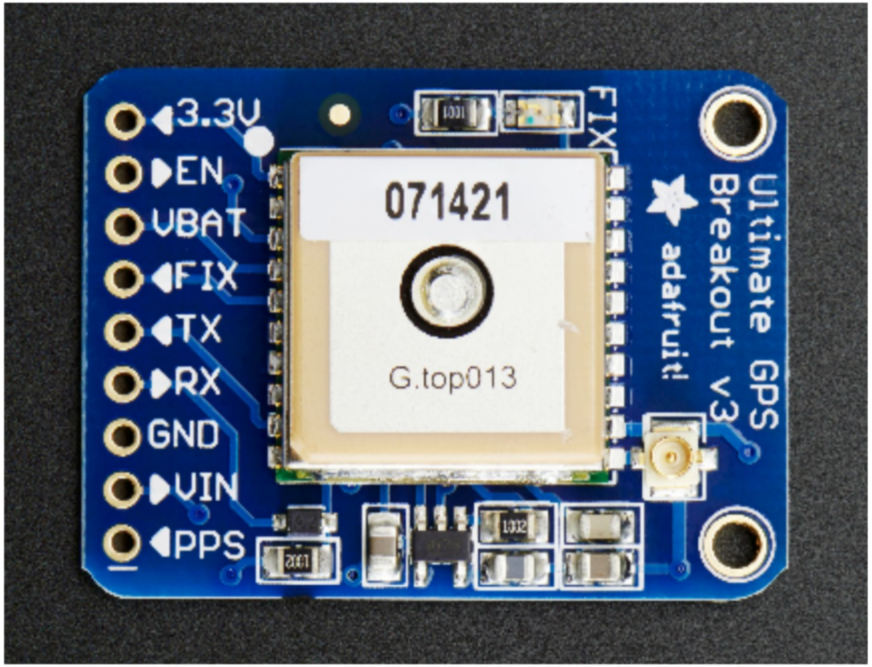
1980s
PSN-8 Manpack GPS Receiver

-- National Museum of American History,
Smithsonian Institution

-- prc68.com, Brooke Clarke

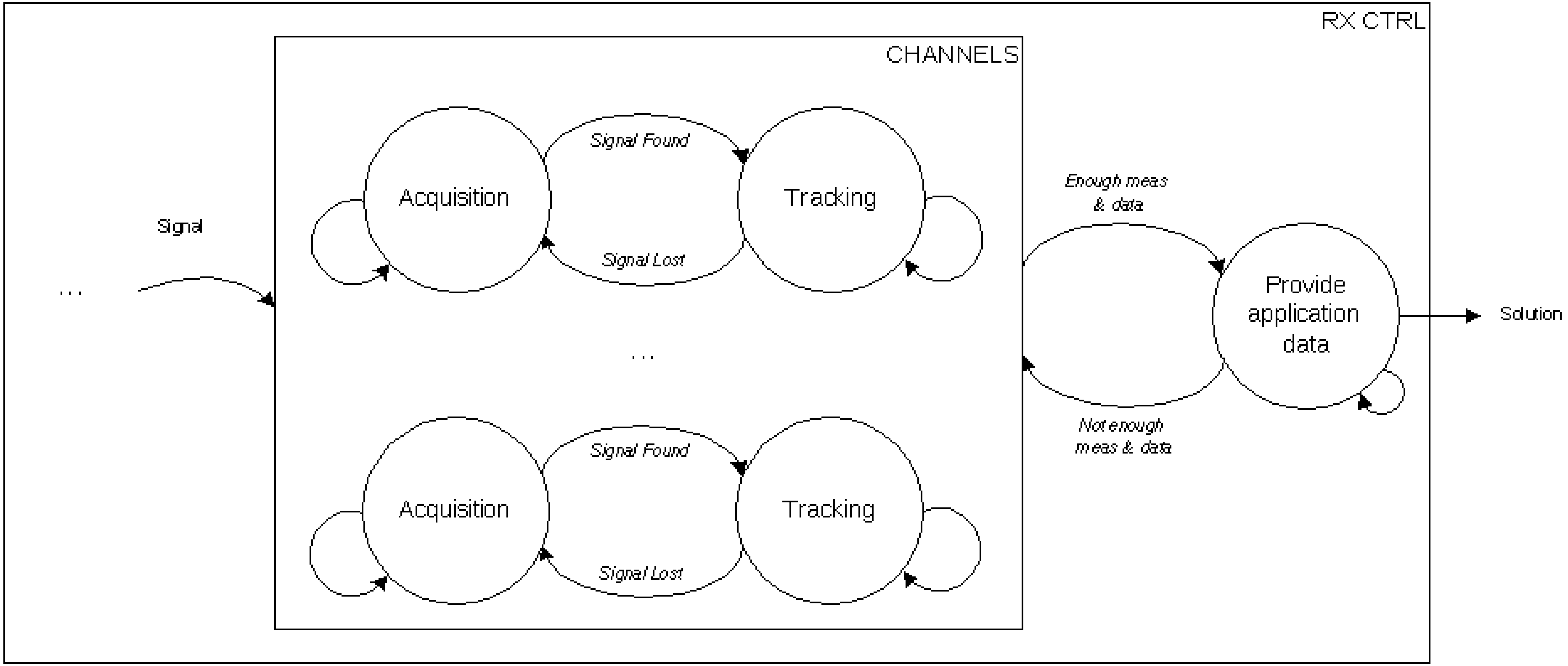
2010s

Garmin eTrex® Touch 35



Adafruit GPS module with patch antenna

Internals


RX CTRL
CHANNELS
Signal
...
Acquisition
Signal Found
Tracking
Signal Lost
...
Acquisition
Signal Found
Tracking
Signal Lost
Enough meas
& data
Provide
application
data
Not enough
meas & data
Solution

Convert GPS Equations to Real Simple Example

GPS equations for calculating location
using four satellites in three dimensions ...

$$\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2} + ct_B = d_1$$
$$\sqrt{(x_0 - x_2)^2 + (y_0 - y_2)^2 + (z_0 - z_2)^2} + ct_B = d_2$$
$$\sqrt{(x_0 - x_3)^2 + (y_0 - y_3)^2 + (z_0 - z_3)^2} + ct_B = d_3$$
$$\sqrt{(x_0 - x_4)^2 + (y_0 - y_4)^2 + (z_0 - z_4)^2} + ct_B = d_4$$

... to SimpleWorld location equation
using two stations in one dimension

$$x_2 - ((x_2 - x_1) - (t_2 - t_1))/2 = x_0$$

and figure out the current time

$$(x_0 - x_1) + t_1 = t_0$$

Convert GPS Equations to Real Simple Example

GPS equations for calculating location

$$\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2} + ct_B = d_1$$
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$$\sqrt{(x_0 - x_3)^2 + (y_0 - y_3)^2 + (z_0 - z_3)^2} + ct_B = d_3$$
$$\sqrt{(x_0 - x_4)^2 + (y_0 - y_4)^2 + (z_0 - z_4)^2} + ct_B = d_4$$

from three dimensions and time ...

Convert GPS Equations to Real Simple Example

$$\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2 + (z_0 - z_1)^2} + ct_B = d_1$$
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$$\sqrt{(x_0 - x_3)^2 + (y_0 - y_3)^2 + (z_0 - z_3)^2} + ct_B = d_3$$
$$\sqrt{(x_0 - x_4)^2 + (y_0 - y_4)^2 + (z_0 - z_4)^2} + ct_B = d_4$$

... to one dimension and time
also correction for local time error is zero and can be removed

Convert GPS Equations to Real Simple Example

$$\sqrt{(x_0 - x_1)^2} = d_1$$
$$\sqrt{(x_0 - x_2)^2} = d_2$$

square and square root almost cancel either other ...
the result can be positive or negative

$$\pm(x_0 - x_1) = d_1$$
$$\pm(x_0 - x_2) = d_2$$

change distance to speed of signal times time difference

$$\pm(x_0 - x_1) = c(t_0 - t_1)$$
$$\pm(x_0 - x_2) = c(t_0 - t_2)$$

c is one in SimpeWorld and can be removed
add t_2 to both sides of second equation

$$\pm(x_0 - x_1) = t_0 - t_1$$
$$t_2 \pm (x_0 - x_2) = t_0$$

replace t_0 in first equation with second equation

$$\pm(x_0 - x_1) = (t_2 \pm (x_0 - x_2)) - t_1$$

Convert GPS Equations to Real Simple Example

$$\pm(x_0 - x_1) = (t_2 \pm (x_0 - x_2)) - t_1$$

shuffle parts around

$$\pm(x_0 - x_1) = t_2 - t_1 \pm (x_0 - x_2)$$

$$\cdots$$

$$\pm(x_0 - x_1) - \mp(x_0 - x_2) = t_2 - t_1$$

restricting the order to $x_1 < x_0 < x_2$ solidifies the signs

$$+(x_0 - x_1) - -(x_0 - x_2) = t_2 - t_1$$

$$\cdots$$

$$x_0 - x_1 + x_0 - x_2 = t_2 - t_1$$

$$\cdots$$

$$2x_0 - (x_1 + x_2) = t_2 - t_1$$

$$\cdots$$

$$-2x_0 + (x_1 + x_2) = -(t_2 - t_1)$$

$$-2x_0 + (x_1 + x_2) = -(t_2 - t_1)$$

$x_2 = 2x_2 - x_2$

$$-2x_0 + x_1 + (2x_2 - x_2) = -(t_2 - t_1)$$

$$\cdots$$

$$2x_2 + -2x_0 - x_2 + x_1 = -(t_2 - t_1)$$

$$\cdots$$

$$2(x_2 - x_0) - x_2 + x_1 = -(t_2 - t_1)$$

$$\cdots$$

$$2(x_2 - x_0) = x_2 - x_1 - (t_2 - t_1)$$

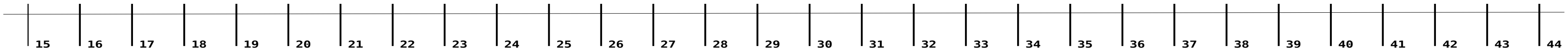
$$\cdots$$

$$x_2 - x_0 = ((x_2 - x_1) - (t_2 - t_1))/2$$

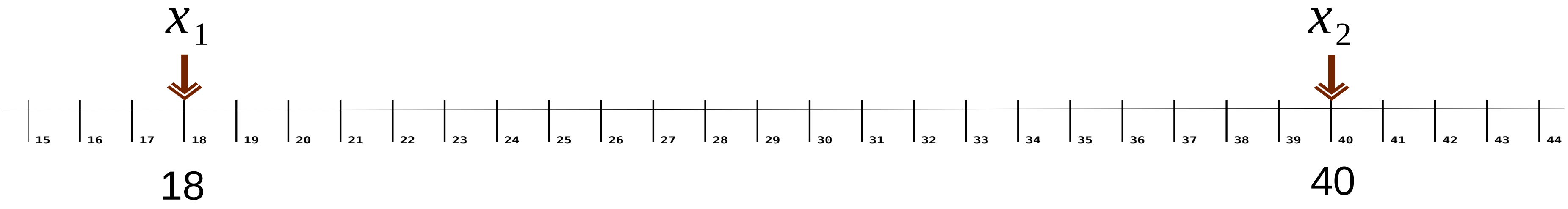
$$\cdots$$

$$x_2 - ((x_2 - x_1) - (t_2 - t_1))/2 = x_0$$

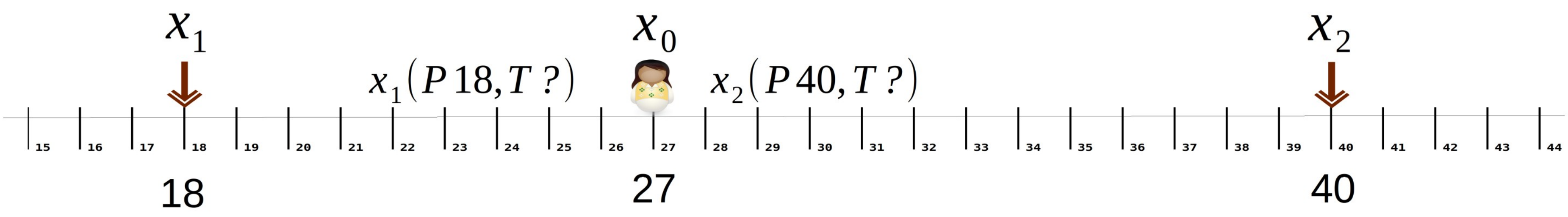
SimpleWorld Example



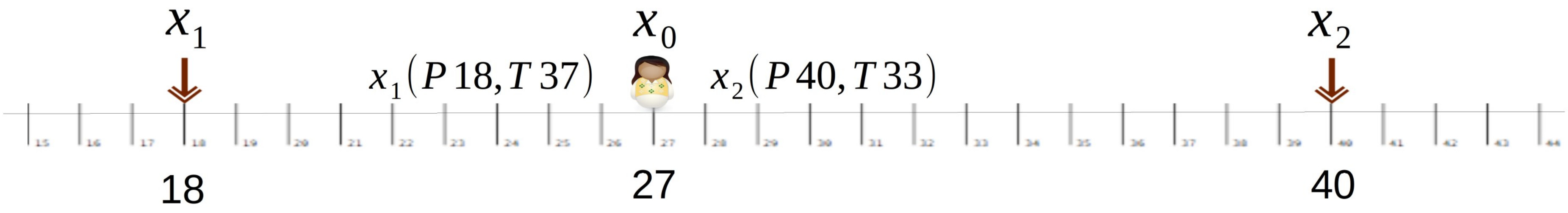
SimpleWorld Example



SimpleWorld Example

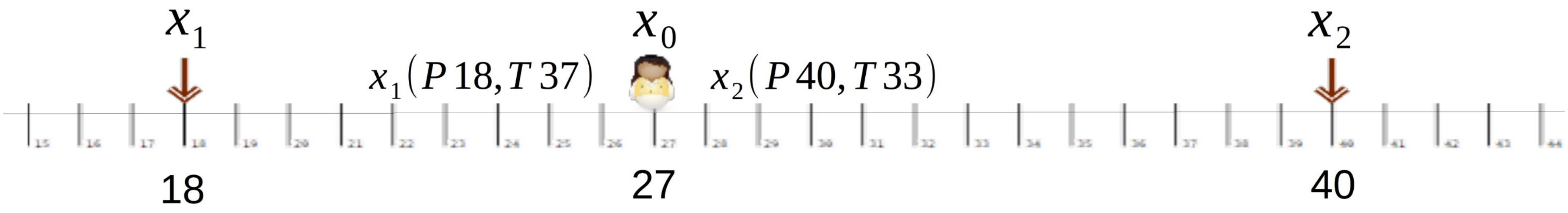


SimpleWorld Example




```
calculate message timestamps
  (x2 - X0) - (X0 - X1) = diff
  (40 - 27) - (27 - 18)
  (  13   ) - (   9   )
            4           = diff
pick a time T2, add diff to get T1
  T2 + diff = T1
  33 +  4
     37     = T1
```

SimpleWorld Example

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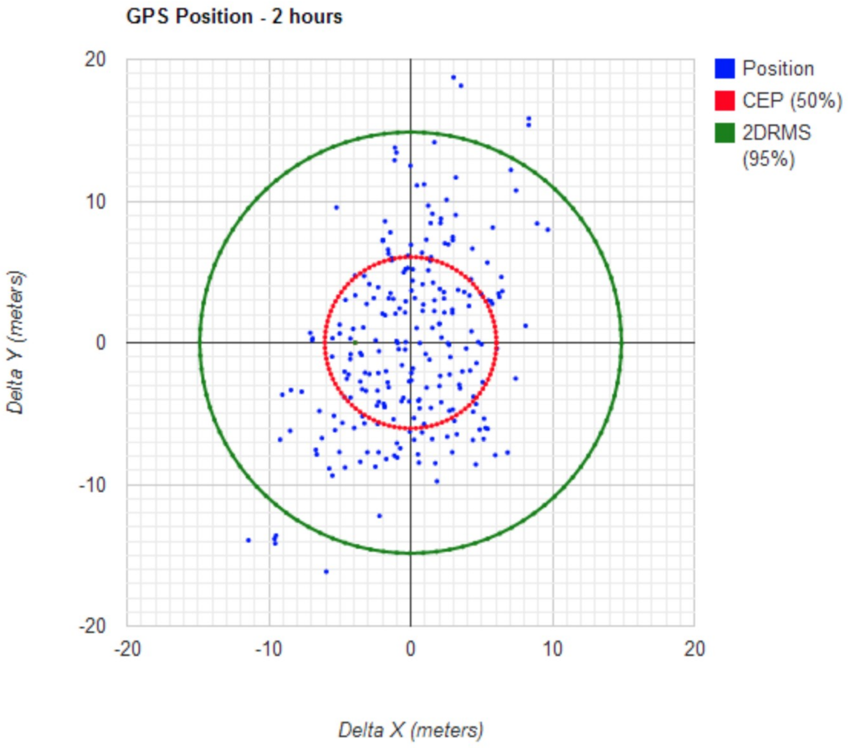
  (  13   ) - (   9   )
            4           = diff
pick a time T2, add diff to get T1
  T2 + diff = T1
  33 +  4
     37     = T1
```

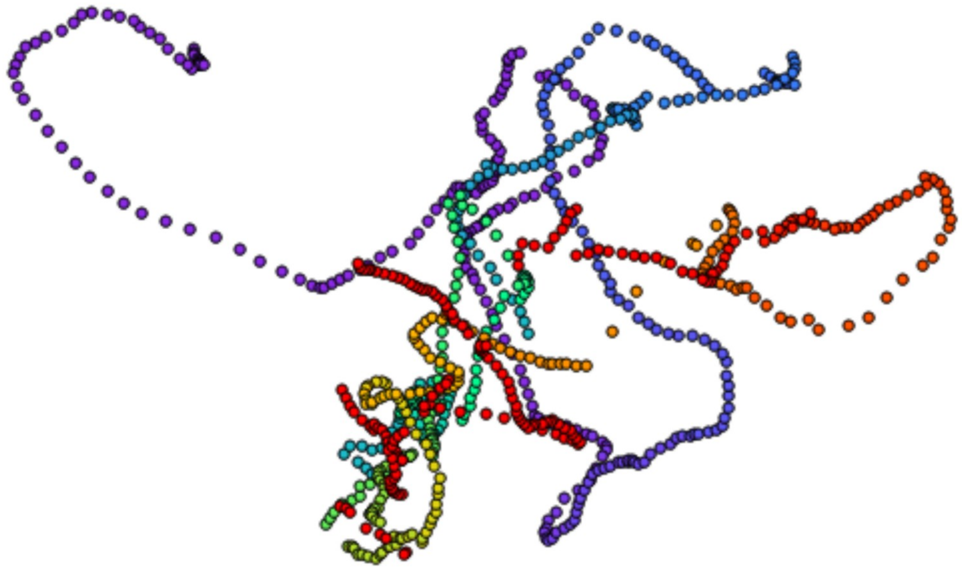
```
calculate location X0
  X2 - ( (X2 - X1) - (T2 - T1) ) / 2 = X0
  40 - ( (40 - 18) - (33 - 37) ) / 2
  40 - ( (  22    ) - (  -4    ) ) / 2
  40 - (           26            ) / 2
  40 -               13
                   27                = X0

calculate time T0
  ( X0 - X1 ) + T1 = T0
  ( 27 - 18 ) + 37
  (    9    ) + 37
          46       = T0
```

Accuracy

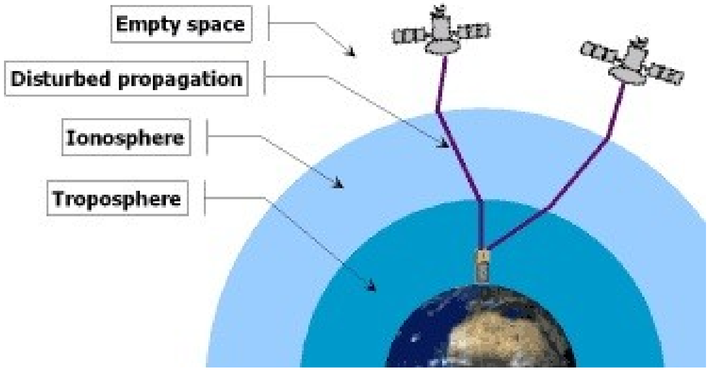


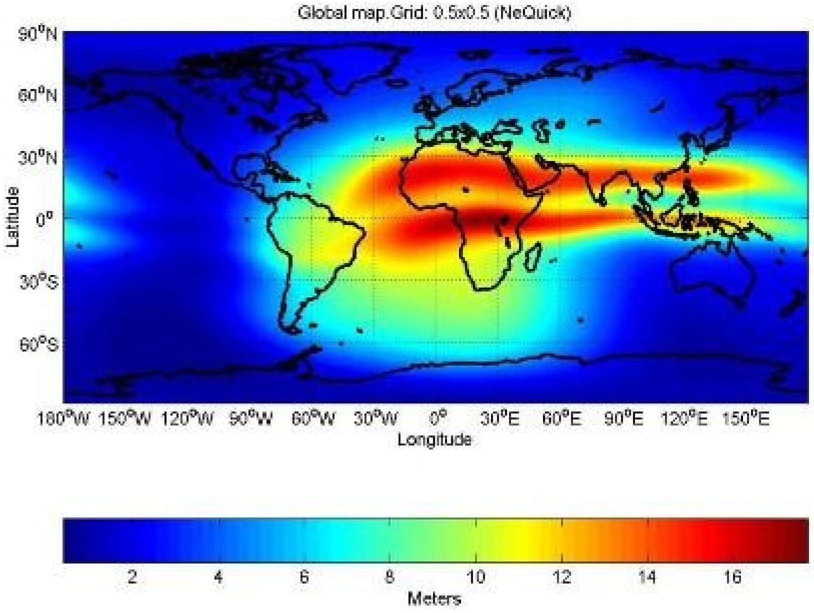
Michael Coyle, blog.oplopanax.ca



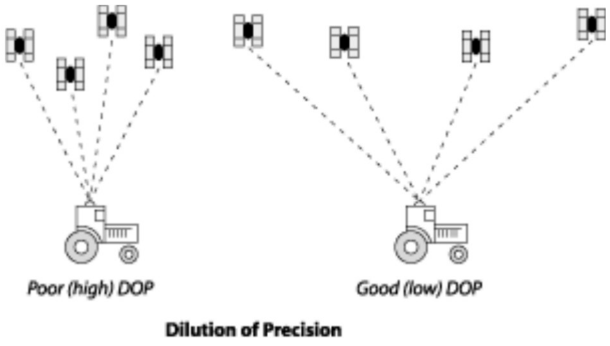
Garmin GPS18 USB device at a stationary location,
750 points, one point per second over 12.5 minutes, about
6 meters east/west by 4 meters north/south
www.georeference.org

Error Sources
Ionosphere – up to 7 meters



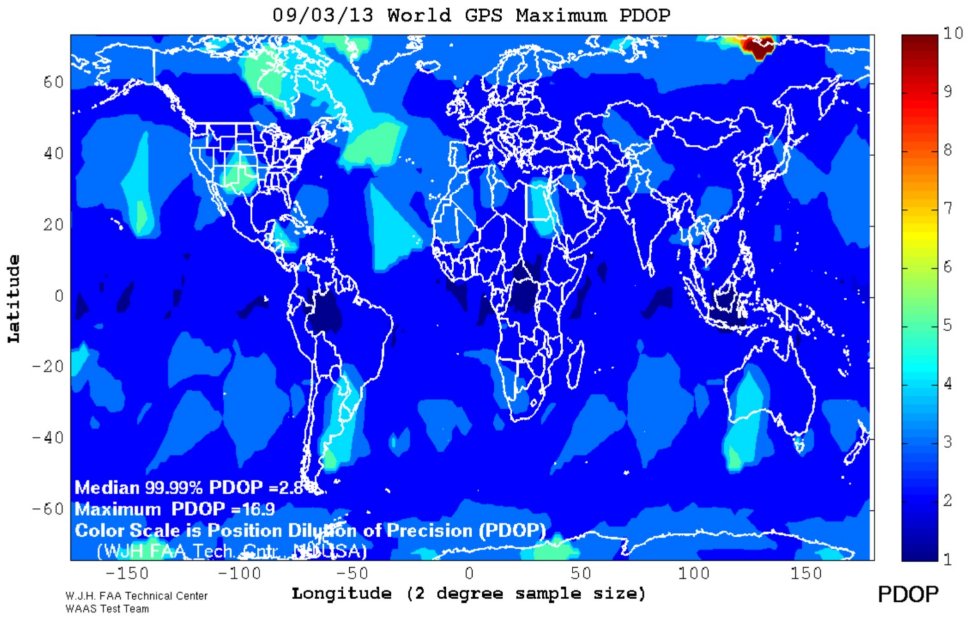


Error Sources
Dilution of Precision (DOP)
up to 6.7 meters

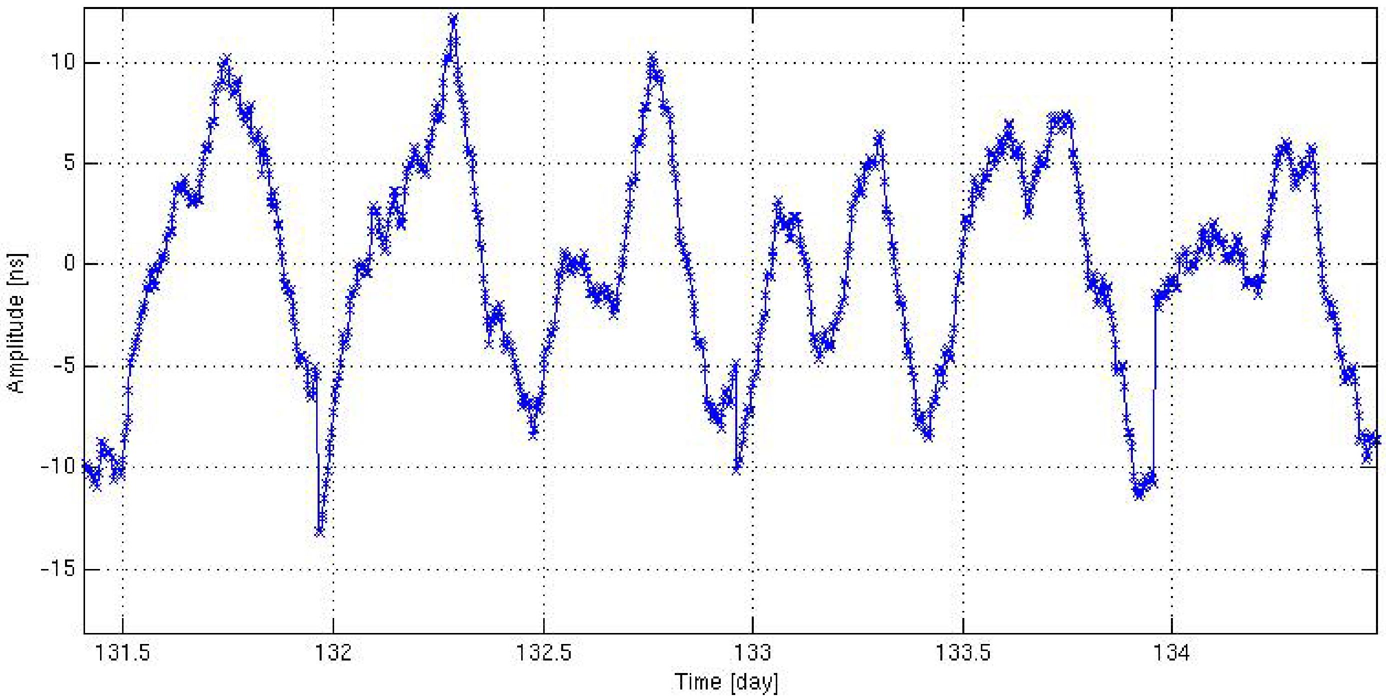


EDOP - east (X) DOP
NDOP - north (Y) DOP
HDOP - horizontal DOP
VDOP - vertical DOP
PDOP - position (3D) DOP
TDOP - time DOP
GDOP - geometric DOP

$$EDOP^2 + NDOP^2 = HDOP^2$$
$$HDOP^2 + VDOP^2 = PDOP^2$$
$$PDOP^2 + TDOP^2 = GDOP^2$$

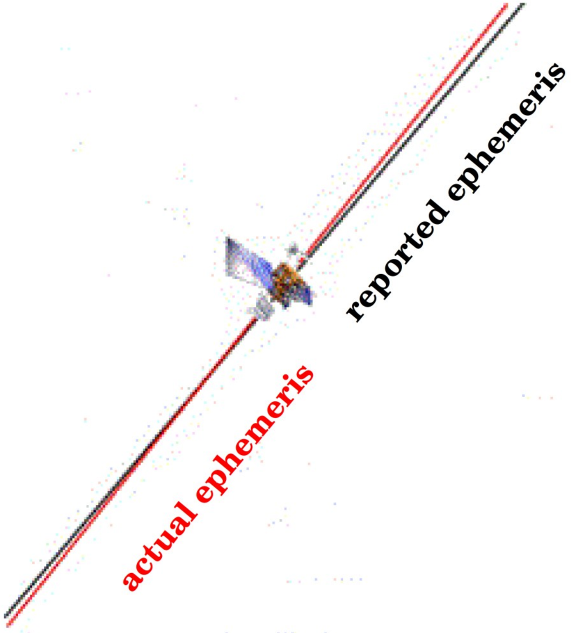


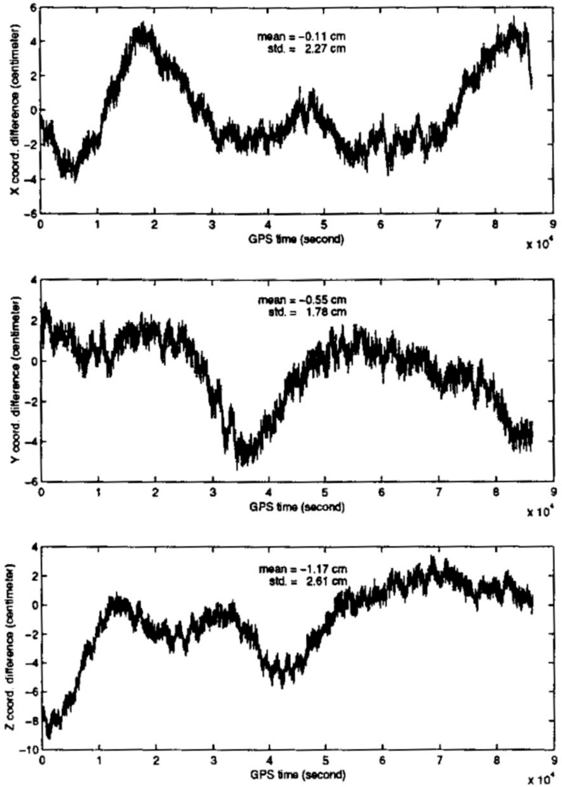
Error Sources
Satellite Clock – up to 3.6 meters



c x 10 nanoseconds = 3 meters

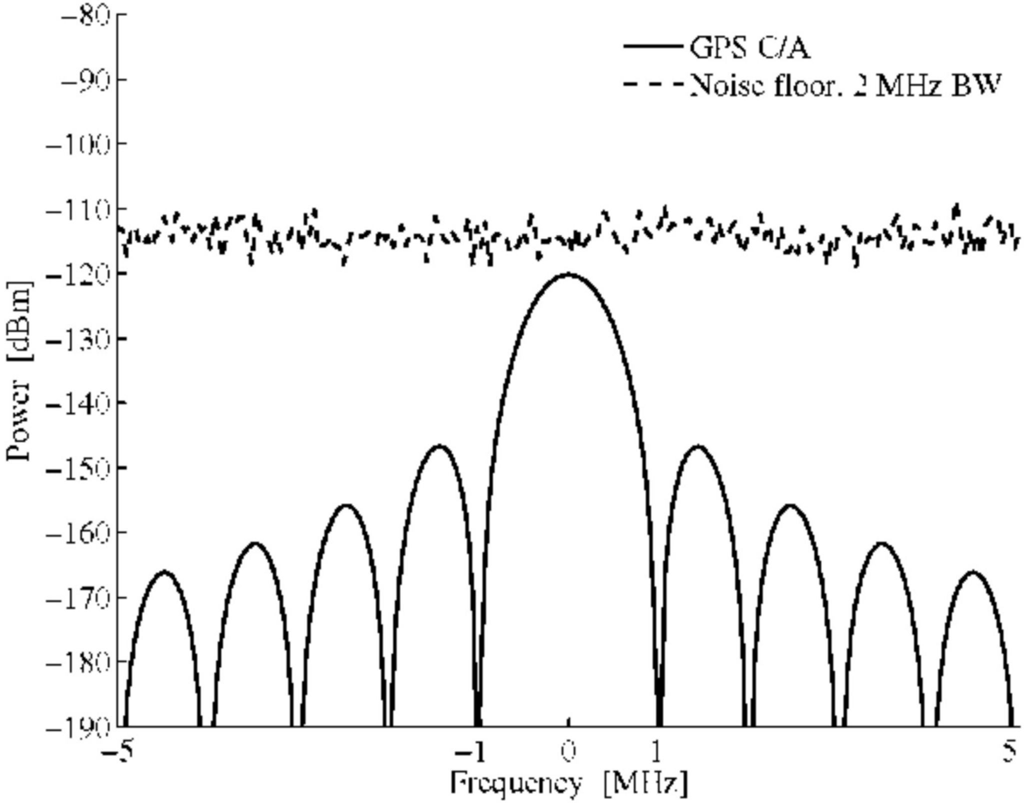
Error Sources
Satellite Orbit – up to 2.5 meters



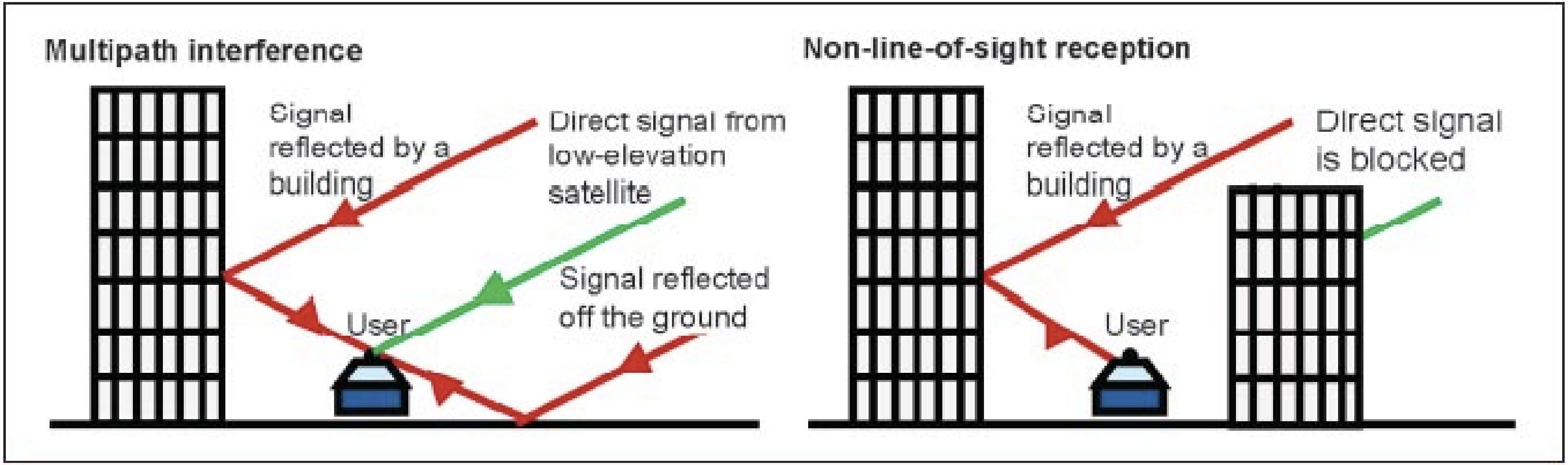


Differences between the interpolated IGS orbit and the JPL 30-sec. orbit for PRN07

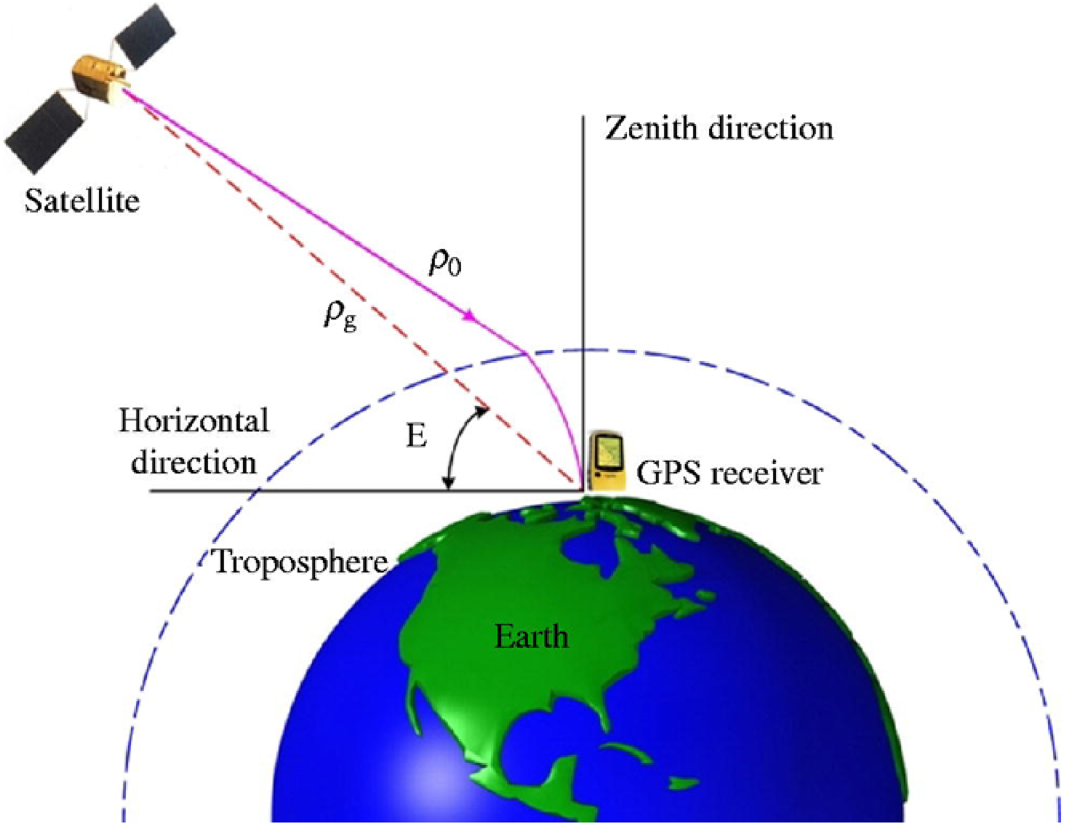
Error Sources
Receiver Noise – up to 1.5 meters



Error Sources
Multipath – up to 1.2 Meters



Error Sources
Troposphere – up to 0.7 Meters



Error Sources
Relativity – 0 Meters

- Compensated for in satellite
  - Satellite speed, Special Relativity
    - Clock slow 7 microseconds/day
  - Satellite in Earth's gravity, General Relativity
    - Clock fast 45.9 microseconds/day
- Compensated for in receiver
  - Sagnac effect, Earth's rotation, east-west error
    - tens of meters

Improving Accuracy

1. Clear open view of the sky, clear to the horizons
2. Multiple signals from each satellite
3. Reference stations
   - Wide Area Augmentation System (WAAS)
   - Differential GPS (DGPS)
   - Surveying

Questions?

Satellite Navigation

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An International
Association of Technology
& Computer User Groups